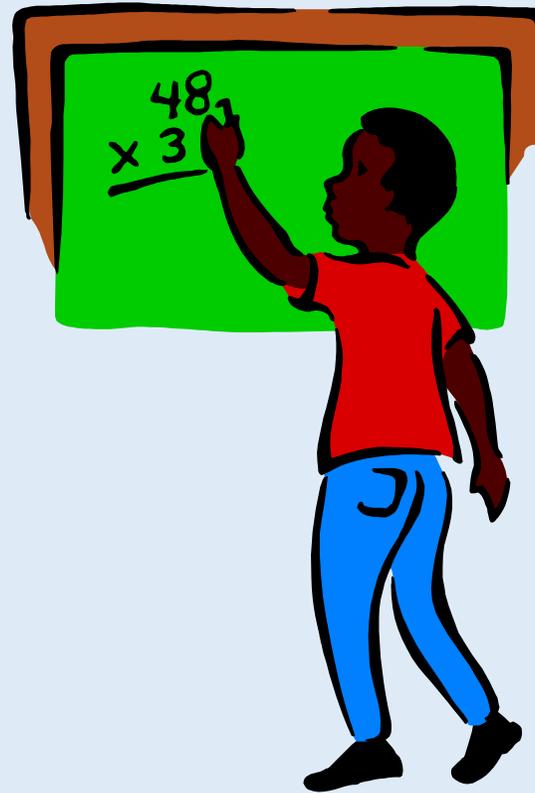


Session 3

Multiplication



Multiplication



Concrete resources

Place value counters

Dienes

Place value charts

Arrays

Multiplication squares

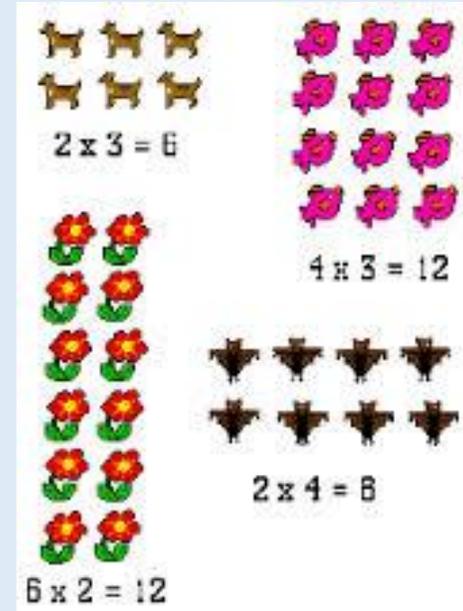
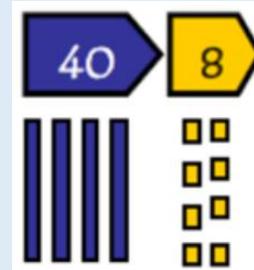
100 square

Number lines

Blank number lines

Counting stick

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

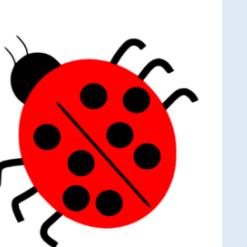


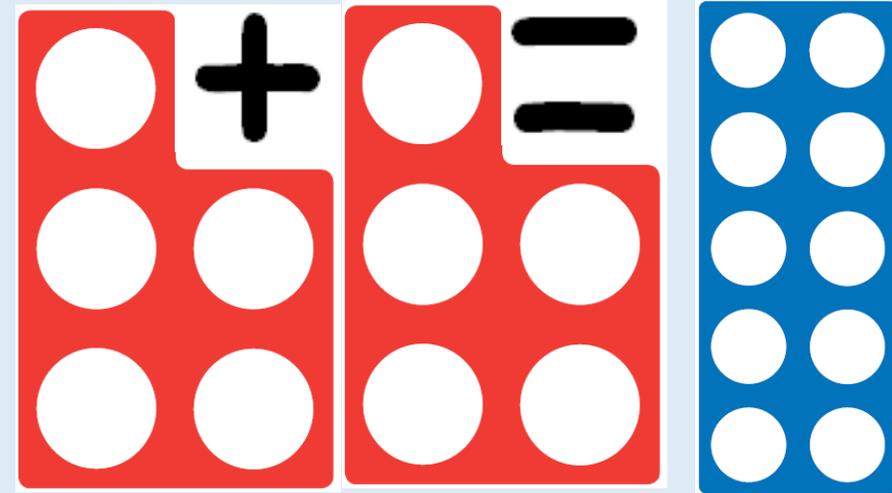
multiplication **product**
once, twice, three times
double **groups of**
repeated addition **lots of**
array, row, column **multiply**
times **multiple**

Multiplication: Reception

Early learning goal statutory requirement:
✓ They solve problems, including doubling, halving and sharing.

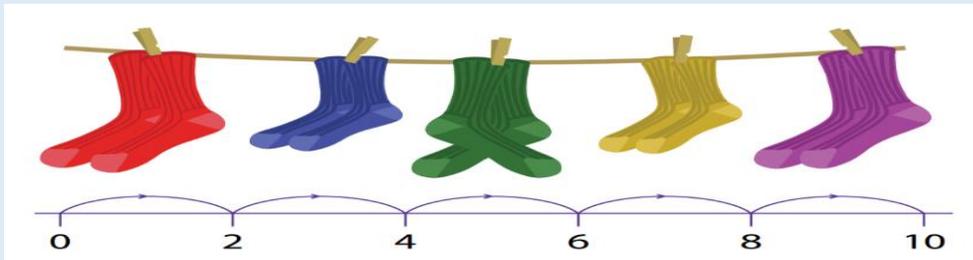
Use pictorial representations and concrete resources to double numbers to 10.

			
$1 + 1 = 2$	$2 + 2 = 4$	$3 + 3 = 6$	$4 + 4 = 8$

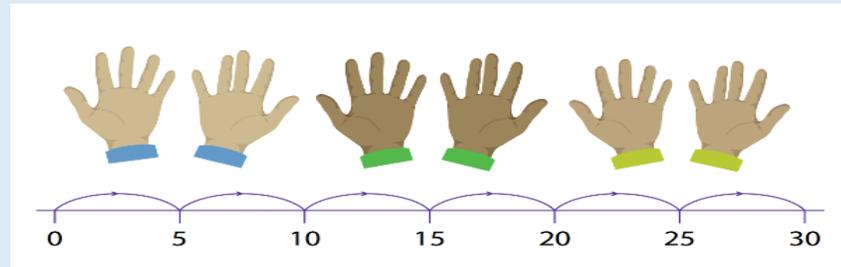


Two red ten-frames are shown side-by-side. The first has a plus sign (+) above it, and the second has an equals sign (=) above it. To the right is a blue ten-frame.

Use concrete sources, role play, stories and songs to begin counting in twos, fives and tens.



A number line from 0 to 10 with arrows indicating jumps of 2. Above the line are five pairs of socks: red, blue, green, yellow, and purple.



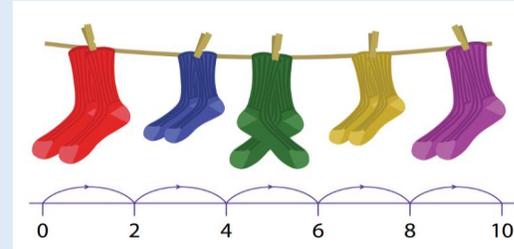
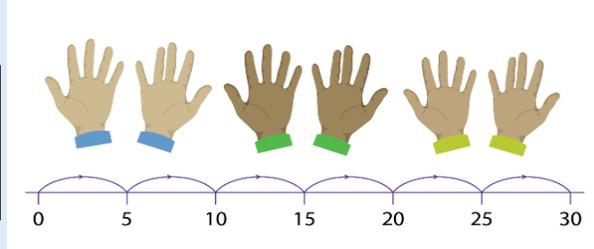
A number line from 0 to 30 with arrows indicating jumps of 5. Above the line are six hands of different colors: light skin, tan, brown, dark brown, olive, and yellow.

Multiplication: Year 1

Year 1 statutory requirement:

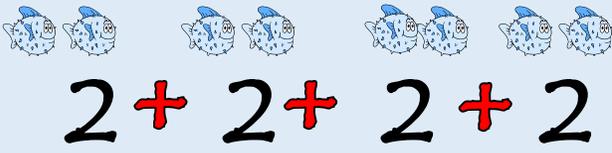
✓ Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Count in twos, fives and tens using practical resources, role play, stories and songs.

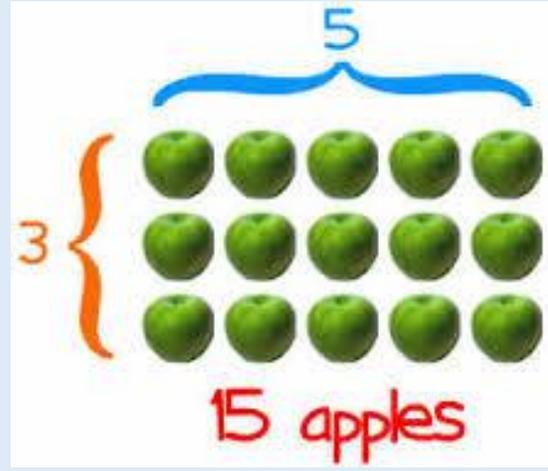
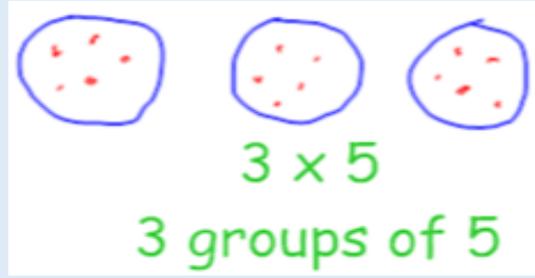


Use arrays

Understand multiplication as repeated addition – use concrete objects to support understanding.



Use pictorial representations



Multiplication: Year 2

Year 2 statutory requirement:

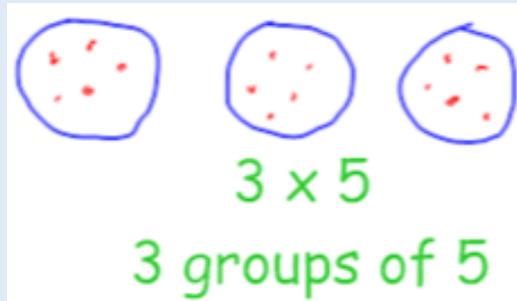
- ✓ Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- ✓ Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs.
- ✓ Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- ✓ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Further develop understanding multiplication as repeated addition.

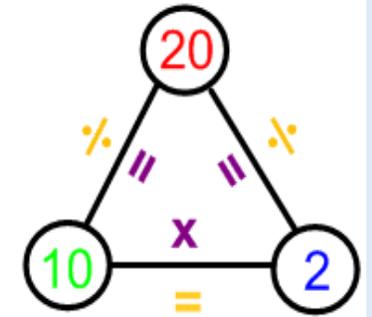
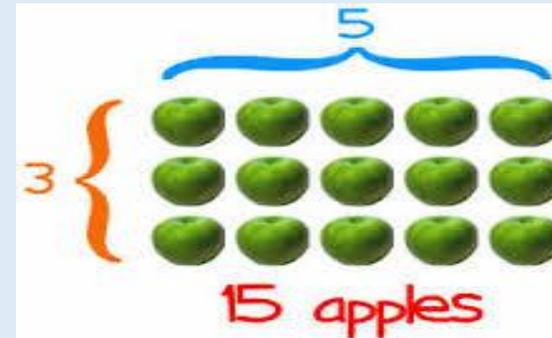


$5 + 5 + 5$
or
 5×3

Use pictorial representations

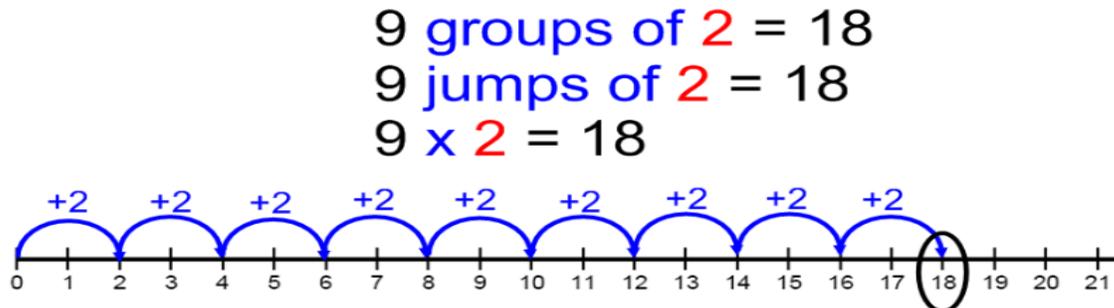


Use arrays



Recall multiplication and division facts for 2, 5, 10

Model and bridge link from repeated addition to solving multiplication problems using a number line.



Beginning to multiply

When we begin to multiply we start by counting in steps of 2, 3 across a numberline.



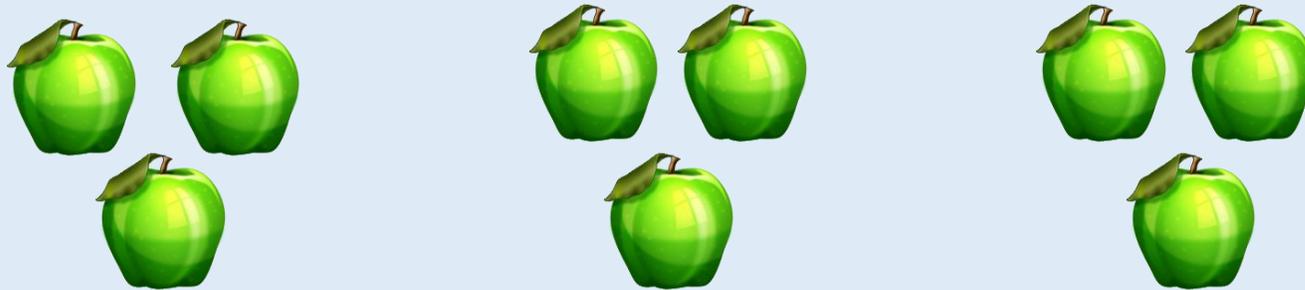
Mental Strategies for Multiplication

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10×10
- apply times tables facts to similar calculations such as 7×5 \longrightarrow 70×5 , 70×50 , 700×5 or 700×50 using their knowledge of place value;
- partition numbers into multiples of Hundreds, Tens and Units
- add two or more single-digit numbers, multiples of 10 and 100 and combinations of whole numbers using the column method.
- understand the language of multiplication including lots of, groups of, times, multiply, product

Written methods for Multiplication

Initially multiplication is introduced as 'repeated addition' using vocabulary such as 'lots of' or 'groups of' and real objects or pictures.



3 lots of 3 = 9 leading to $3 \times 3 = 9$

Solving Multiplication Calculations – before written methods

Use of arrays to solve simple problems

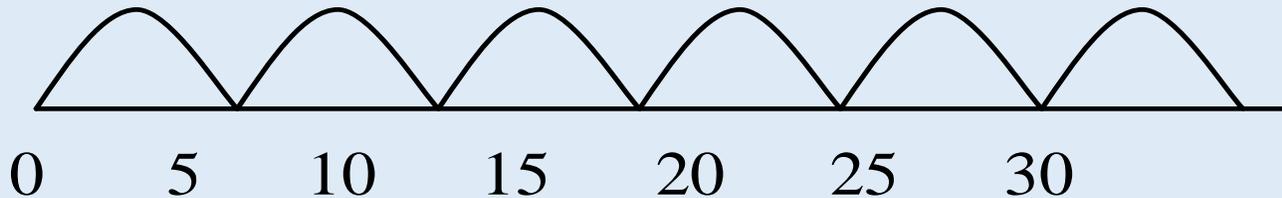
• • • •

• • • • $4 \times 2 = 8$

$2 \times 4 = 8$

Use a numberline to multiply

$$5 \times 6 = 30$$

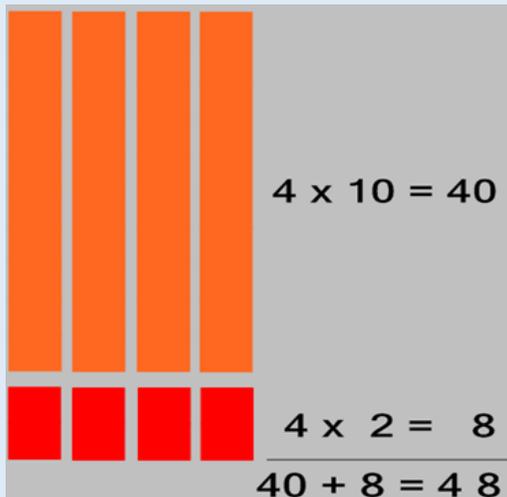




Develop recall of multiplication facts (alongside inverse of the corresponding division facts).

x	3	4	8	x	4	?	?
5				?	8	6	10
6				6	24	18	30
4				?	32	24	40

$12 \times 4 = 48$



Multiplication: Year 3

Year 3 statutory requirements:

- ✓ Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.
- ✓ Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- ✓ Solve problems, including missing number problems, involving multiplication including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Use concrete resources to develop conceptual understanding of the compact method introduced in Year 4.

x	10	2
4		

OR

x	10	2
4	40	8

$$\begin{array}{r}
 10 + 2 \\
 \times \quad 4 \\
 \hline
 8 \\
 40 \\
 \hline
 48
 \end{array}$$



The yellow ribbon is 4 times as long as the red ribbon. What is its length?

Written methods for Multiplication

In KS2 the aim is that children develop rapid recall of all times tables to 12×12 and can use an efficient written method for

- two-digit by one-digit multiplication by the end of Year 4 (TU x U)
- two-digit by two-digit multiplication by the end of Year 5 (TU x TU)
- three-digit by two-digit multiplication by the end of Year 6 (HTU x TU)

Written methods for Multiplication

Year 3: Mental multiplication using partitioning

This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens. This can look like.....

$$\begin{array}{r}
 43 \\
 40 + 3 \\
 \downarrow \quad \downarrow \\
 240 + 18 = 258
 \end{array}
 \times 6$$

$$\begin{aligned}
 14 \times 3 &= (10 + 4) \times 3 \\
 &= (10 \times 3) + (4 \times 3) = 30 + 12 = 42
 \end{aligned}$$

Written methods for Multiplication

Year 3: The Grid Method

This links directly to the mental method. It is an alternative way of recording the same steps. It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products. For TU x TU, the partial products in each row are added, and then the two sums at the end of each row are added to find the total product

×	7
30	210
8	56
	266

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

Written methods for Multiplication

The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products in preparation for the standard method.

	30 + 8
×	7
	210
	56
	266

	50	6	
×	20	7	
	1000	350	1350
	120	42	162
			1512
			1

Over to you!

Have a go at solving these multiplications using the grid method.

$$65 \times 8$$

$$74 \times 45$$

$$92 \times 53$$

Multiplication: Year 4

Year 4 statutory requirement:

- ✓ Recall multiplication and division facts for multiplication tables up to 12×12
- ✓ Use place value, known and derived facts to multiply and divide mentally, including: multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
- ✓ Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Build on learning from Year 3 and model how grid method and/or expanded method links to compact short multiplication.

x	30	6
4		



$$\begin{array}{r}
 30 + 6 \\
 \times 4 \\
 \hline
 24 \\
 + 120 \\
 \hline
 144
 \end{array}$$

$$\begin{array}{r}
 ^2 36 \\
 \times 4 \\
 \hline
 144
 \end{array}$$

Develop recall of multiplication facts (alongside the inverse of the corresponding division facts).



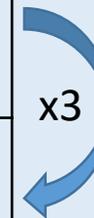
2 eggs
150g flour
180g sugar

Use knowledge of times tables to solve scaling problems.

Susie wants to bake 12 cupcakes people.
The ingredients given are for four cupcakes.
How much flour will she need?



<u>Cupcakes</u>	<u>Flour</u>
4	150g
12	900g



Written methods for Multiplication

Year 3/4: Expanded short multiplication

The next step is to represent the method in a column format, but showing the working. Attention should be drawn to the links with the grid method above. Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38×7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3×7 should be stressed. Most children should be able to use this expanded method for $TU \times U$ by the end of Year 4.

$$\begin{array}{r}
 30 + 8 \\
 \times \quad 7 \\
 \hline
 210 \\
 \quad 56 \\
 \hline
 266
 \end{array}$$

$$30 \times 7 = 210$$

$$8 \times 7 = 56$$



$$\begin{array}{r}
 38 \\
 \times \quad 7 \\
 \hline
 210 \\
 \quad 56 \\
 \hline
 266
 \end{array}$$

Written methods for Multiplication

Year 3/4: Expanded short multiplication

The same steps can be used when introducing TU x TU.

$$\begin{array}{r}
 56 \\
 \times \underline{27} \\
 1000 \\
 120 \\
 350 \\
 \underline{42} \\
 1512 \\
 1
 \end{array}$$

$$\begin{array}{l}
 50 \times 20 = 1000 \\
 6 \times 20 = 120 \\
 50 \times 7 = 350 \\
 6 \times 7 = 42
 \end{array}$$



$$\begin{array}{r}
 56 \\
 \times \underline{27} \\
 1120 \\
 \underline{392} \\
 1512 \\
 1
 \end{array}$$

$$\begin{array}{l}
 56 \times 20 \\
 56 \times 7
 \end{array}$$

Over to you!

Have a go at solving these multiplications using the expanded short method.

$$32 \times 8$$

$$56 \times 15$$

$$78 \times 37$$

Written methods for Multiplication

When ready: Short multiplication

The expanded method is eventually reduced to the standard method for short multiplication. The recording is reduced further, with carry digits recorded below the line. If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.

$$\begin{array}{r} 38 \\ \times \quad 7 \\ \hline 266 \\ \hline 5 \end{array}$$

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage

Multiplication: Year 5

Year 5 statutory requirements:

- ✓ Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.
- ✓ Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Build on learning from Year 4 and use concrete resources if needed to multiply numbers up to 4 digits by **one digit** using compact short multiplication.

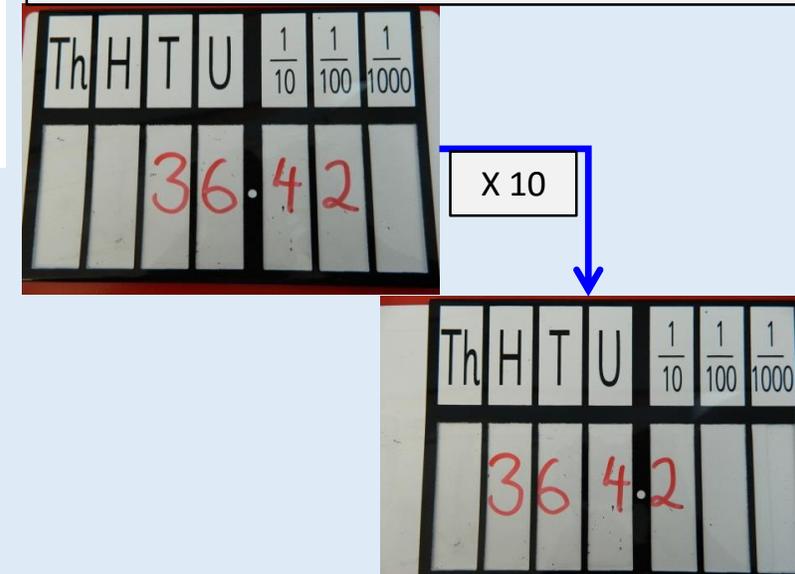
$$\begin{array}{r}
 643 \times 8 \\
 ^3 ^2 \\
 643 \\
 \times 8 \\
 \hline
 5144
 \end{array}$$

x	600	40	3
50	30,000	2,000	150
4	2,400	160	12

Reinforce the connection between the grid method to multiply numbers up to 4 digits by **two digit** using long multiplication.

$$\begin{array}{r}
 ^2 ^1 \\
 ^1 ^1 \\
 643 \\
 \times 54 \\
 \hline
 2572 \\
 +32150 \\
 \hline
 34722
 \end{array}$$

To multiply by 10, 100, 1000 children should use place value charts to show that the digit moves a column (s) to the left. The value of the digit is increasing by 10, 100 or 1000 times.



Written methods for Multiplication

Year 5: Long multiplication

This is applied to TU x TU as follows.

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 1120 \\ \quad 392 \\ \hline 1512 \\ 1 \end{array}$$
$$\begin{array}{l} 56 \times 20 \\ 56 \times 7 \end{array}$$

The carry digits in the partial products of $56 \times 20 = 120$ and $56 \times 7 = 392$ are usually carried mentally.

The aim is for most children to use this long multiplication method for TU x TU by the end of Year 5.

Multiplication: Year 6

Year 6 statutory requirements:

- ✓ Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- ✓ Multiply one-digit numbers with up to two decimal places by whole numbers.

Build on learning from Year 5 multiplying numbers using compact short multiplication and long division.

$$\begin{array}{r}
 643 \times 8 \\
 \overset{3}{6} \overset{2}{4} 3 \\
 \times 8 \\
 \hline
 5144
 \end{array}$$

$$\begin{array}{r}
 21 \\
 11 \\
 643 \\
 \times 54 \\
 \hline
 2572 \\
 \overset{1}{3} 2150 \\
 \hline
 34722
 \end{array}$$

Use compact short multiplication to multiply decimal number by whole number.

$$\begin{array}{r}
 23 \\
 7.68 \\
 \times \quad 4 \\
 \hline
 30.72
 \end{array}$$

Written methods for Multiplication

In Year 6, children apply the same steps to multiply HTU x TU

×	20	9	
200	4000	1800	5800
80	1600	720	2320
6	120	54	174
			8294
			1

Start with the grid method, asking the children to estimate their answer first.

$$\begin{array}{r}
 286 \\
 \times 29 \\
 \hline
 4000 \\
 1600 \\
 120 \\
 1800 \\
 720 \\
 54 \\
 \hline
 8294 \\
 1
 \end{array}$$

$200 \times 20 = 4000$
 $80 \times 20 = 1600$
 $6 \times 20 = 120$
 $200 \times 9 = 1800$
 $80 \times 9 = 720$
 $6 \times 9 = 54$

This expanded method is cumbersome, so there is plenty of incentive to move on to a more efficient method.

$$\begin{array}{r}
 286 \\
 \times 29 \\
 \hline
 5720 \\
 2574 \\
 \hline
 8294 \\
 1
 \end{array}$$

286×20
 286×9

Children who are already secure with multiplication for TU x U and TU x TU should have little difficulty in using the same method for HTU x TU.

<https://gfletchy.com/progression-videos/>